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COMMON FIXED POINT THEOREMS OF ITERATIONS FOR WEAK CONTRACTION IN CONE METRIC SPACES

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ABSTRACT
In the present paper we prove common fixed point theorems in complete cone metric Spaces for weak contraction which generalize and extend some well-known previous results of [9].

KEYWORDS: Cone metric spaces, Common fixed point Picard iteration, Weak Contraction.

INTRODUCTION
Fixed point theory as an important branch of nonlinear functional analysis theory has been applied in many disciplines, see for instance [6-8]. By replacing the real numbers with an ordered Banach space, Huang and Zhang [1] defined cone metric spaces and proved some fixed point theorems of contractions on cone metric spaces. Since several authors have studied the fixed point problem of nonlinear mappings in cone metric spaces see for instance [1],[2],[3],[4],[5].

The purpose of this paper is to extend and prove some common fixed point of general contractions in cone metric spaces. Our results generalize and the respective theorems of [9].

PRELIMINARY NOTES
First, we recall some standard notations and definitions in cone metric spaces properties [1].

Definition 2.1 [1]: Let E be a real Banach space and P be a subset of E. P is called a cone if and only if:
(i) P is closed, non – empty and $P \neq \{0\}$,
(ii) $ax + by \in P$ for all $x, y \in P$ and non – negative real number a, b;
(iii) $x \in P$ and $-x \in P \Rightarrow x = 0 \Leftrightarrow P \cap (-P) = \{0\}$.
Given a cone $P \subset E$, we define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x \ll y$ if $y - x \in \text{int } P$, int P denotes the interior of P. Note that, it is clear if $a \leq b, c \leq d$, then $a + c \leq b + d$, and for $\mu \in R, \mu \geq 0, \mu a \leq \mu b$.
The cone P is called normal if there is a number K > 0 such that $x, y \in E, 0 \leq x \leq y$ implies $\| x \| \leq K \| y \|$.
In following we always suppose E is a Banach space. P is a cone in E with $intP \neq \phi$ and $\leq$ is partial ordering with respect

Definition 2.2 [1]: Let X be a non – empty set. Suppose the mapping $d: X xX \rightarrow E$ satisfies
(i) $0 < d(x, y)$ for all $x, y \in X$ and $d(x,) = 0$ if and only if $x = y$;
(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(iii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$. Then d is called a cone metric on X, and (X, d) is called a cone metric space.

Definition 2.3 [9]: Let (X, d) be a cone metric space, T a self map of X. Let $x_{2n+1} = f(T, x_{2n})$ be some iteration procedure. Suppose that $F(T)$, the fixed point set of, is nonempty and that x_{2n} to a point $p \in F(T)$. Let $\{y_n\} \subset X$, and define $\epsilon_{2n} = d\left(y_{2n+1}, f(T, y_{2n})\right)$. If $\lim_{n\to\infty} \epsilon_{2n} = 0$ implies $\lim_{n\to\infty} y_{2n} = p$, then $x_{2n+1} = f(T, x_{2n})$ is called stable with respect T.

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MAIN RESULTS

The following theorem is an extension of theorem 2.1 and 3.2 of [9].

Theorem 3.1 Let X be a nonempty complete cone metric space. $T_1, T_2 : X \times X \to X$ be any two self maps on X such that

$$d(T_1x, T_2y) \leq h(x,y) + Ld(y, T_1x) \ldots\ldots\ldots\ldots (3.1.1)$$

For all $x, y \in X$, where h is some real number in [0, 1], and L is some real number in $(0,\infty)$. Then T_1 and T_2 have a common fixed point in X .

Proof. Let $x_0 \in X$ and $n \geq 1$, Let $\{x_{2n}\}$ be a sequence generated by the following Picard iteration

$$x_{2n} = T_1\, x_{2n-1} \text{ and } x_{2n+1} = T_2\, x_{2n}$$

Now Taking $x = x_{2n-1}$ and $y = x_{2n}$. Then we get

$$d\left(T_1x_{2n-1,}\, T_2x_{2n}\right) \leq hd\left(x_{2n-1,}x_{2n}\right).$$

This implies that $d\left(x_{2n,}\, x_{2n+1}\right) \leq hd\left(x_{2n-1,}x_{2n}\right)$.

In general

$$d\left(x_{2n,}\, x_{2n+1}\right) \leq hd\left(x_{2n-1,}x_{2n}\right)$$
$$\leq h^2 d(x_{2n-1}, x_{2n-2}) \leq \ldots\ldots\ldots\ldots \leq h^{2n} d(x_1, x_0)$$

For $n \geq m$, we have

$$d\left(x_{2n,}x_{2m}\right) \leq d(x_{2n}, x_{2n-1}) + d(x_{2n-1}, x_{2n-2}) + \ldots\ldots + d(x_{2m+1}, x_{2m})$$
$$\leq (h^{2n-1} + h^{2n-2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots + h^{2m}) d(x_{1,}x_0)$$
$$= \frac{h^{2m}}{1-h}\, d\left(x_{1,}x_0\right)$$

For a given $c \in E$ with $0 \leq c$, that is, $c \in intP$, there exist $B(0,r)$ such that c+ $B(0,r) \dot{\subseteq} P$, where $B(0,r) = \{x \in E, \|x\| \leq r\}$. But there exist a positive number N such that $\frac{h^{2m}}{1-h}\, d\left(x_{1,}x_0\right) \in B(0,r)$ for all $m > N$. Therefore, we have

$$d\left(x_{2n,}x_{2m}\right) \leq \frac{h^{2m}}{1-h}\, d\left(x_{1,}x_0\right) \ll c, for\ all\ m > N.$$ This implies that $\{x_{2n}\}$ is a Cauchy sequence and is convergent because of the completeness of X. We denote $p = \lim_{n\to\infty} x_{2n.}$ Notice that

$$d(p, T_1p) \leq d(p, x_{2n+1}) + d\left(x_{2n+1,}T_1p\right)$$
$$d(p, T_1p) = d(p, x_{2n+1}) + d(T_1x_{2n,}\; T_1p\;)$$
$$\leq d(p, x_{2n+1}) + hd\left(x_{2n,}\, p\right) + Ld(p, T_1x_{2n}$$
$$= (1+L)\, d(p,\; x_{2n+1}) + hd(x_{2n}, p) \to 0.$$

This implies that $\lim_{n\to\infty}(p, T_1p) = 0$. Hence $T_1\, p = p$ .Therefore p is a fixed point of T_1.

Now q is another fixed point of T_1. Then we have

$$d(p,q) = 0 \text{ implies that } p = q$$

Therefore, the fixed point of $T_1$is unique. Similarly, it can be established that $T_2p = p$. Hence $T_1p = p = T_2p$. Thus p is the common fixed point of T_1 and T_2.

Theorem 3.2 Let X be a nonempty complete cone metric space. $T_1, T_2 : X \times X$ be any two self - maps on X such that

$$d(T_1x, T_2y) \leq h(x,y) + Ld(x, T_2y) \ldots\ldots\ldots\ldots (3.2.1)$$

For all $x, y \in X$, where h is some real number in [0, 1], and L is some real number in $(0,\infty)$. Then T_1 and T_2 have a common fixed point in X .

Proof: The proof of this theorem is similar to the proof of the theorem 3.1.

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